The dynamics of vortex structures and states of the current in plasma-like fluids and the electrical explosion of conductors: IV. Model of the first splitting stage of an exploding conductor

1995 J. Phys. A: Math. Gen. 28 1789

(http://iopscience.iop.org/0305-4470/28/7/004)

J. Phys. A: Math. Gen. 28 (1995) 1789–1797. Printed in the UK

The dynamics of vortex structures and states of the current in plasma-like fluids and the electrical explosion of conductors: IV. Model of the first splitting stage of an exploding conductor

Nickolas B Volkov and Alexander M Iskoldsky

Russian Academy of Science, Ural Division, Institute of Electrophysics, 34 Komsomolskaya St, Ekaterinburg 620219, Russia

Received 14 November 1994

Abstract. This paper is a continuation of a series of papers (1993 *J. Phys. A: Math. Gen.* **26** 6635, 6649, 6667), and makes use of the same assumptions. The paper presents a model of the first stage of the spatial scale splitting of hydrodynamic vortex structures. It is shown that this model is one of a first-order non-equilibrium phase transition, which leads to the forming of a heterogeneous low-conducting current state of the exploding conductor. As this takes place, its dynamics are determined by three interacting order parameters. As a result the exploding conductor is broken down into metallic particles with the size of the conductor diameter. It is shown that the transition from a high- to low-conducting state by the splitting of the spatial scale is energetically more profitable than the transition by excitation of current vortex structures. Parameters of the model, which describe the formation stage of hydrodynamic and current vortex structures, are identified by experimental results for the electrical explosion of conductors. The problems which emerged while bringing together the models describing these stages are also discussed.

1. Introduction

This paper is the fourth part of our series [1–3] where the second-order non-equilibrium phase transition (NPT) model was constructed [1]; the dynamics of hydrodynamic and current large-scale structures was studied in plasma-like current-carrying fluids [2], and a comparison was made of the computer experiment results and the results of experiments on the electrical explosion of conductors (EEC) [3]. The aim of the present work consists of constructing and studying a model of the first stage of splitting the hydrodynamic vortex structures (according to the hypothesis expressed in [1] the spatial scale splitting is caused by doubling the excitation wavenumber (k)). Towards the end of this stage the exploding (current-carrying) conductor is broken down into transverse strata, and the electric current is interrupted. As shown below (see section 3) the model of the first stage of the spatial scale splitting is a first-order NPT as opposed to the second-order NPT studied in [1–3].

2. Model of the non-equilibrium phase transition

An analogy between initial stages of the turbulence nucleation and the EEC, as deduced in [4], is applied below to construct a model of the first stage of the spatial scale splitting.

Here, as in [1,4], we shall restrict our consideration to an incompressible conducting fluid the transport coefficients of which—the electric conductivity (σ) and the shear viscosity (η)—are constant. This enables us to focus on the dynamical nature of the NPT discussed here.

A set of magnetohydrodynamic (MHD) equations is an input to an infinite-dimensional model for constructing low-dimensional NPT models. In our case it becomes

$$(\nabla \cdot \boldsymbol{u}) = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -{\rho_0}^{-1}\nabla P + (4\pi\rho_0)^{-1}[[\nabla, \boldsymbol{H}], \boldsymbol{H}] + \nu\Delta\boldsymbol{u} \tag{2}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{H} = (\boldsymbol{H} \cdot \nabla)\boldsymbol{u} + \nu_{\mathrm{m}}\Delta\boldsymbol{H} \tag{3}$$

where $\boldsymbol{u}$, P, $\boldsymbol{H}$ and ρ_0 are the velocity, the pressure, the magnetic field strength and the mass density, respectively; $\nu = \eta{\rho_0}^{-1}$ and $\nu_{\mathrm{m}} = c^2(4\pi\sigma)^{-1}$ are the kinematic viscosity and the magnetic viscosity, respectively; Δ is the Laplacian; c is the vacuum light velocity and ∇ is the nabla operator. From equation (1) it follows that $\boldsymbol{u} = [\nabla, \boldsymbol{A}]$ where $\boldsymbol{A}$ is the vector potential of the velocity, which is known to be accurate within the gradient of a scalar function. (Below, the Coulomb gauge of the vector potential is applied: $(\nabla \cdot \boldsymbol{A}) = 0$.) We assume, using azimuthal symmetry, that the magnetic field strength $\boldsymbol{H}$ and the vector potential of the velocity $\boldsymbol{A}$ are only one azimuthal component: $\boldsymbol{H} = (0, H(r,z,t), 0)$ and $\boldsymbol{A} = (0, \psi(r,z,t), 0)$. In this case from (1) it follows that $\boldsymbol{u} = (u_r(r,z,t), 0, u_z(r,z,t)) = \left(-\frac{\partial\psi}{\partial z}, 0, \frac{\partial(r\psi)}{r\partial r}\right)$. The magnetic field strength at the conductor boundary equals $H(r_{\mathrm{e}}, z, t) = H_0 r_0 i(t)(I_0 r_{\mathrm{e}}(z,t))^{-1}$, where $H_0 = 2I_0(cr_0)^{-1}$ and $r_{\mathrm{e}}(z,t) = r_0(1 + D(z,t){r_0}^{-1})$; r_0 and I_0 are the initial conductor radius and the characteristic electric current; $D(z,t)$ is the distance of the moving conductor radius from the initial one.

Let us write (1)–(3) in a dimensionless form, using as the base sizes the electric current I_0, the initial conductor radius r_0 and the magnetic viscosity ν_{m} (the term is which break down the azimuthal symmetry are disregarded, as the threshold of the bending excitations is well beyond that of the excitations discussed below):

$$\begin{aligned}\frac{\partial(\Delta\psi - \psi r^{-2})}{\partial t} &= -\frac{\partial(\psi, \Delta\psi - \psi r^{-2})}{\partial(r,z)} + Rs\frac{H}{r}\frac{\partial H}{\partial z} \\ &\quad + s\left[\Delta\left(\Delta\psi - \psi r^{-2}\right) - r^{-2}\left(\Delta\psi - \psi r^{-2}\right)\right]\end{aligned} \tag{4}$$

$$\frac{\partial H}{\partial t} = -\frac{\partial(\psi, H)}{\partial(r,z)} + \Delta H - \frac{H}{r^2} \tag{5}$$

where

$$\frac{\partial(a,b)}{\partial(r,z)} = \frac{\partial(ra)}{r\partial r}\frac{\partial b}{\partial z} - \frac{\partial a}{\partial z}\frac{\partial(rb)}{r\partial r}$$

and $R = {H_0}^2{r_0}^2(2\pi\rho_0\nu_{\mathrm{m}}\nu)^{-1} = {v_{\mathrm{A}}}^2{r_0}^2(\nu_{\mathrm{m}}\nu)^{-1} = {Pe_{\mathrm{m}}}^2 s^{-1}$ is the magnetic Rayleigh number and $v_{\mathrm{A}} = H_0(2\pi\rho_0)^{-1/2}$ is the Alfven velocity; $Pe_{\mathrm{m}} = v_{\mathrm{A}} r_0 {\nu_{\mathrm{m}}}^{-1}$ is the magnetic Péclet number and $s = \nu{\nu_{\mathrm{m}}}^{-1}$. Equations (4) and (5) agree accurately with the replacement of the Cartesian coordinate system by the cylindrical one and the replacement of the heat conduction equation for the diffusion of the magnetic field with Saltzman equations [5] in the theory of the Benard effect [6] in the case of the assumed azimuthal symmetry.

The solutions of (4) and (5) are sought in the form: $H(r,z,t) = H_1(r,z,t) + h(r,z,t)$ and $\psi(r,z,t) = \psi_1(r,z,t) + \varphi(r,z,t)$. For the unperturbed solutions H_1 and ψ_1 we take the following expressions: $H_1(r,z,t) = I(t)r(1+\delta)^{-2}$ and $\psi_1(r,z,t) = c_s(z,t)r$, where $\delta = D{r_0}^{-1}$ and $I = i(t){I_0}^{-1}$, since our interest is only in constructing the low-mode NPT model, giving a qualitative description of the transition to the low-conducting current state of a conductor. The expression for H_1 provides the homogeneous distribution of an electric current density on the cross section of the conductor. The expression for ψ_1 gives an approximate description of the heat expansion of the conductor, providing the nearly low behaviour of the radial velocity.

We obtain two sets of equations for calculating θ, δ, φ and h, restricting oneself to the terms, which are linear in the excitations, and using the expressions for ψ_1 and H_1:

$$\frac{\partial\theta}{\partial t} = 2\theta\frac{\partial c_s}{\partial z} - 2c_s\frac{\partial\theta}{\partial z} - 2RsI^2(1+\delta)^{-5}\frac{\partial\delta}{\partial z} + s\frac{\partial^2\theta}{\partial z^2} \qquad \text{where} \quad \theta = \frac{\partial^2 c_s}{\partial z^2} \tag{6}$$

$$\frac{\partial\delta}{\partial t} = -2c_s\frac{\partial\delta}{\partial z} - 0.5(1+\delta)\frac{\partial c_s}{\partial z} + \frac{\partial^2\delta}{\partial z^2} - 3(1+\delta)^{-1}\left(\frac{\partial\delta}{\partial z}\right)^2 + (1+\delta)(2I)^{-1}\frac{\mathrm{d}I}{\mathrm{d}t} \tag{7}$$

and

$$\begin{aligned}\frac{\partial(\Delta\varphi - \varphi r^{-2})}{\partial t} = &-\frac{\partial(\varphi, \Delta\varphi - \varphi r^{-2})}{\partial(r,z)} - 2c_s\frac{\partial(\Delta\varphi - \varphi r^{-2})}{\partial z} + r\frac{\partial c_s}{\partial z}\frac{\partial(\Delta\varphi - \varphi r^{-2})}{\partial r} \\ &-\frac{\partial r\varphi}{\partial r}\frac{\partial\theta}{\partial z} + \theta\frac{\partial\varphi}{\partial z} + Rs\left(-2Ih(1+\delta)^{-3}\frac{\partial\delta}{\partial z} + I(1+\delta)^{-2}\frac{\partial h}{\partial z}\right) \\ &+s\big(\Delta(\Delta\varphi - \varphi r^{-2}) - r^{-2}(\Delta\varphi - \varphi r^{-2})\big)\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial h}{\partial t} = &-\frac{\partial(\varphi, h)}{\partial(r,z)} + 2I(1+\delta)^{-3}\frac{\partial r\varphi}{\partial r}\frac{\partial\delta}{\partial z} + I(1+\delta)^{-2}\frac{\partial\varphi}{\partial z} - 2c_s\frac{\partial h}{\partial z} \\ &+r\frac{\partial c_s}{\partial z}\frac{\partial h}{\partial r} + \Delta h - hr^{-2}.\end{aligned} \tag{9}$$

Let us assume the following boundary conditions (l is the conductor length): $\delta(0) = \delta(l) = c_s(0) = c_s(l) = \frac{\partial c_s(0)}{\partial z} = \frac{\partial c_s(l)}{\partial z} = \theta(0) = \theta(l) = 0$ and $\varphi(0,z,t) = \varphi(1+\delta,z,t) = \varphi(r,0,t) = \varphi(r,l,t) = h(0,z,t) = h(1+\delta,z,t) = 0$.

The dynamical system processes (equations (6)–(9)) are broken down into two stages. In the first stage, which is equal to condition $r_e = r_0 =$ constant, the large-scale vortex hydrodynamic and current structures are developed. Their dynamics are determined by a set of three ordinary differential equations. This set can be obtained from (8) and (9), using the following permutation [1]:

$$\varphi(r,z,t) = \sqrt{2}\frac{1+(\pi k/g_1)^2}{k}X(t)\sin(\pi kz)J_1(g_1 r) \tag{10}$$

$$h(r,z,t) = (\pi r_1)^{-1}\big(\sqrt{2}Y(t)\cos(\pi kz)J_1(g_1 r) - 2Z(t)J_0(g_1 r)J_1(g_1 r)\big) \tag{11}$$

where $g_1 = 3.831\,71$ corresponds to the first zero of the Bessel function $J_1(x)$, $r_1 = RR_c{}^{-1}$ is the control parameter, $R_c = 64g_1{}^2\pi^2(b^2(4-b))^{-1}$ is the critical Rayleigh number, and $b = 4(1+(\pi kg_1{}^{-1})^2)^{-1}$:

$$\dot{X} = s(-X + IY) \tag{12}$$

$$\dot{Y} = \pi g_1{}^{-1}X\big(-Z + \pi g_1{}^{-1}r_1 I\big) - Y \tag{13}$$

$$\dot{Z} = -\big(\pi g_1{}^{-1}XY + bZ\big) \tag{14}$$

where the point symbol '$\cdot$' is used to denote the differentiation operator for the dimensionless time $\tau = tt_0{}^{-1}$; $t_0 = br_0{}^2(4g_1{}^2\nu_m)^{-1}$ is the basic time constant. (Below, we denote the dimensionless time 'τ', as usual, by 't'.) Equations (12) and (13) must be added to equations for an external electric circuit and a relation between the conductor voltage drop and the electric current. In the case $r_e = r_0 =$ constant this relation is determined by the following expression [1]:

$$u(t) = u_l(t) + u_r(t) = L_\rho I_0\left(c^2 t_0\right)^{-1}\dot{I} + R_\rho I_0\left(I - a_1 r_1{}^{-1} Z\right) \qquad (15)$$

where L_ρ and $R_\rho = l(\pi r_0{}^2\sigma)^{-1}$ are the initial conductor inductance and the initial conductor resistance, respectively; $a_1 = \pi J_0{}^2(g_1)$.

Let us make a few remarks in preparation for constructing a model for the second stage where the current-carrying conductor is stratified. (Below, these remarks are exploited in the construction of a model for the first splitting stage of the hydrodynamic structures spatial scale.)

As shown in [1, 4], the critical Rayleigh number exhibits a minimum value: $R_{c,\min} = 978$ corresponding to $b = \frac{8}{3}$. As this occurs, the vortex spatial structure with the wavelength $\lambda = r_0 k_0{}^{-1} = 1.159 r_0$ is excited as soon as the control parameter r_1 reaches its critical value (r_*). (The dynamical system (equations (12)–(14)) loses its stability by $r_1 = r_*$.) Figure 1(*a*) shows a field of the hydrodynamic velocity, corresponding to the vortex structure, and (*b*) shows a change of $\omega\omega_m{}^{-1}$ along the conductor axis, where

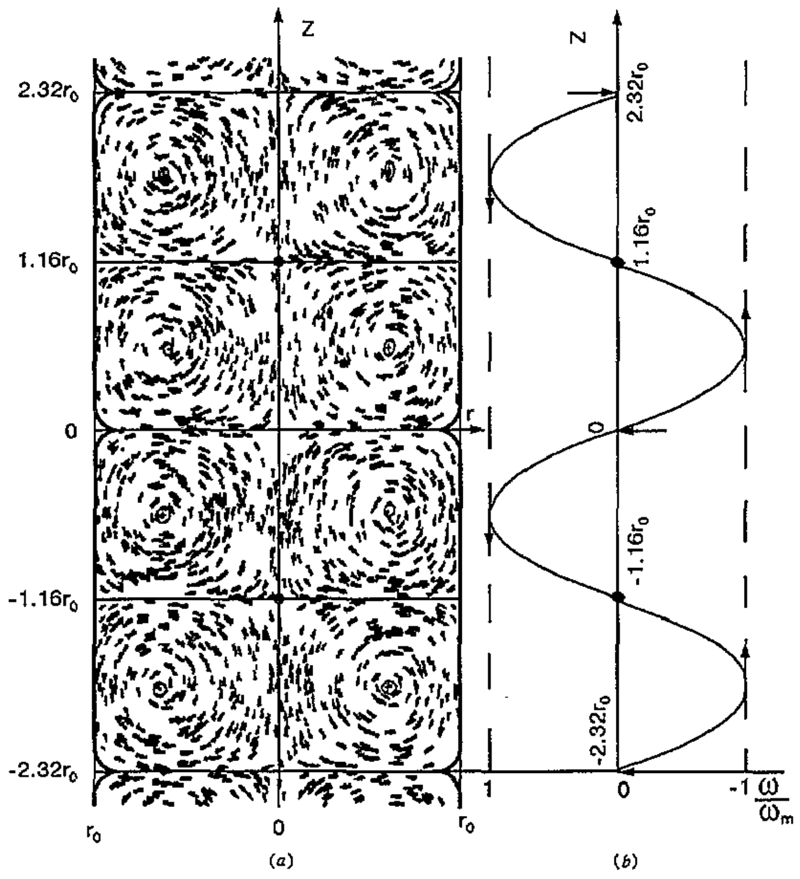

Figure 1. (*a*) The hydrodynamic velocity field and (*b*) the relation $\omega\omega_m{}^{-1}$ as a function of z, where $\omega = \omega_m(t)\sin(\pi k z)$ is the magnitude of the azimuthal component of the vector $O = [\nabla, u]$; $\omega_m = 4\sqrt{2}(bk)^{-1}X(t)J_1(g_1 r_s)$; r_s is a solution of the equation: $\mathrm{d}J_1(g_1 r)/\mathrm{d}r = 0$. (•) Denote locations of Joule-heating sources.

$\omega = \omega_m(t)\sin(\pi k z)$ is the magnitude of the azimuthal component of the vector $\boldsymbol{O} = [\nabla, \boldsymbol{u}]$; $\omega_m = 4\sqrt{2}(bk)^{-1}X(t)J_1(g_1 r_s)$: r_s is the solution of the equation: $\mathrm{d}J_1(g_1 r)/\mathrm{d}r = 0$. This vortex structure presents a system of vortex rings such that its total angular momentum is zero. The vortex rings go along the axis because the direction of motion is decided by the direction of the vector $\boldsymbol{O} = [\nabla, \boldsymbol{u}]$ (in figure 1(*b*) this direction is shown by arrows applied to points where $|\boldsymbol{O}| = |\boldsymbol{O}|_{\max}$, corresponding to $\omega\omega_m{}^{-1} = \pm 1$). Figure 1 shows that a ring slit may spring up between the vortex rings with opposite-directed vectors $\boldsymbol{O}$. The ring slit may be caused by expansion tensions which give rise, as shown in [7], to an excitation of the viscosity liquid surface at points where $z = (2n+1)\lambda$, $n = 0, 1, \ldots, N-1$. ($N = l(2\lambda)^{-1}$ is the number of vortex rings, packed in the conductor length. Below we assume that N has an integer size; direct observations of the stratification of the exploding conductor [8] support this hypothesis.) An analysis of the model (equations (12) and (13)) made in [1] lets us express the hypothesis as a set of spatial scale splittings, performed by the doubling of the excitation wavenumber: $k_0 \to k_1 = 0.5k_0 \to k_2 = 2k_0 \to k_3 = 2k_2 \to \cdots$. (The basis for this hypothesis is shown in figure 1 where we can see pairing of the vortex rings coming from the opposite direction to the vector $\boldsymbol{O}$.)

Let us construct the model of the initial stage of splitting corresponding to the transition $k_0 \to k_1 = 0.5k_0$ ($\lambda_1 = 2\lambda_0 = 2.32r_0$). (This size corresponds well to an experimentally obtained distance between strata [3, 8].) The solution of (6) and (7) is sought in the form of a periodic soliton-like function sufficiently restricted for conservation of the liquid incompressibility:

$$\delta(z,t) = -A(t)\sum_{n=0}^{N}\exp\left(-\left((2n+1)\lambda - z\right)^{2m}\right) = -A(t)F(z) \tag{16}$$

where the parameter 'm' determines the width of the ring slit. (Below, rather than considering ring slits, we consider solitons increasing into the interior of the conductor.) From expressions for the radial velocity 'u_r': $u_r = -\frac{\partial\psi_1}{\partial z} = -r\frac{\partial c_s}{\partial z}$ and the velocity of the conductor surface 'u_e': $u_e = -(1+\delta)\frac{\partial c_s}{\partial z} = \frac{\partial(1+\delta)}{\partial t}$ it follows that

$$\frac{\partial c_s}{\partial t} = -(1+\delta)^{-1}\frac{\partial(1+\delta)}{\partial t}. \tag{17}$$

It can be proved by placing (16) in (7) and granting for (17) that the amplitude A is determined from the following equation:

$$\frac{\mathrm{d}\ln((1-AF)I^{-1})}{\mathrm{d}t} = \frac{4A}{1-AF}\frac{\mathrm{d}A}{\mathrm{d}t}\frac{\mathrm{d}F}{\mathrm{d}z}\left(\int_0^z F(\xi)(1-AF(\xi))^{-1}\mathrm{d}\xi\right) - \frac{A}{1-AF}\frac{\mathrm{d}^2F}{\mathrm{d}z^2} - \frac{3A^2}{(1-AF)^2}\left(\frac{\mathrm{d}F}{\mathrm{d}z}\right)^2. \tag{18}$$

We calculate the amplitude A, applying boundary conditions: $F((2n+1)\lambda) = 1$, $\frac{\mathrm{d}F((2n+1)\lambda)}{\mathrm{d}z} = \frac{\mathrm{d}^2F((2n+1)\lambda}{\mathrm{d}z^2} = 0$, $n = 0, 1, \ldots, N-1$. Then $\frac{\mathrm{d}\ln((1-A)I^{-1})}{\mathrm{d}t} = 0$, from where it follows that $A(t) = 1 - II_*{}^{-1}$. (The critical current I_* corresponds to the nucleus of the soliton, increasing into the conductor, from its surface.) Hence (16) takes the form of

$$\delta(z,t) = -\left(1 - II_*{}^{-1}\right)\sum_{n=0}^{N}\exp\left(-\left((2n+1)\lambda - z\right)^{2m}\right) = -A(t)F(z). \tag{19}$$

Equation (7) with $\delta(z,t)$ in the form of (19) is correct for all $m \geqslant 2$, not only for $z = (2n+1)\lambda$, but also for other z, as $\delta(z,t)$ has the delta-function-like form. (Upper bounds of the parameter m furnish additional information, outside the scope of the present model.)

The following conditions are correct at the soliton peak: for $z = (2n+1)\lambda$, $n = 0, 1, \ldots, N-1$: $\frac{\partial h}{\partial z} = 0$, $\frac{\partial(\Delta\varphi - \varphi r^{-2})}{\partial z} \simeq (\Delta\varphi - \varphi r^{-2})z^{-1}$, $c_s\frac{\partial(\Delta\varphi-\varphi r^{-2})}{\partial z} \simeq (\Delta\varphi - \varphi r^{-2})\frac{\partial c_s}{\partial z}$. It can be proved that by granting these conditions and also that $1+\delta$ is a periodic delta-function-like function, that (8) and (9) are rewritten in the form:

$$\frac{\partial(\Delta\varphi - \varphi r^{-2})}{\partial t} = -\frac{\partial(\varphi, \Delta\varphi - \varphi r^{-2})}{\partial(r, z)} - 2(\Delta\varphi - \varphi r^{-2})\frac{\partial c_s}{\partial z} + \frac{\partial c_s}{\partial z}r\frac{\partial(\Delta\varphi - \varphi r^{-2})}{\partial r} + RsI(1+\delta)^{-2}\frac{\partial h}{\partial z} + s\big(\Delta\big(\Delta\varphi - \varphi r^{-2}\big) - r^{-2}\big(\Delta\varphi - \varphi r^{-2}\big)\big) \tag{20}$$

$$\frac{\partial h}{\partial t} = -\frac{\partial(\varphi, h)}{\partial(r, z)} + I(1+\delta)^{-2}\frac{\partial\varphi}{\partial z} + \frac{\partial c_s}{\partial z}r\frac{\partial h}{\partial r} + \Delta h - hr^{-2}. \tag{21}$$

The solutions of (20) and (21) are sought in the form:

$$\varphi(r, z, t) = \sqrt{2}\,\frac{1 + (\pi k/g_1)^2}{k}X(t)\sin(\pi k z)J_1(g_1 r(1+\delta)^{-1}) \tag{22}$$

$$h(r, z, t) = (\pi r_1)^{-1}\big(\sqrt{2}Y(t)\cos(\pi k z)J_1(g_1 r(1+\delta)^{-1}) - 2Z(t)J_0(g_1 r(1+\delta)^{-1})J_1(g_1 r(1+\delta)^{-1})\big). \tag{23}$$

Let us deduce the set of three ordinary differential equations which determine the temporal behaviour of the interacting amplitudes—order parameters—X, Y and Z, placing (22) and (23) into (20) and (21) and granting that the amplitudes X, Y and Z are defined by processes at the peak of the soliton, increasing into the conductor. It becomes

$$\dot{X} = \frac{4\big(1 + 0.5(\pi k I(g_1 I_*)^{-1})^2\big)}{1 + (\pi k I(g_1 I_*)^{-1})^2}I^{-1}\dot{I}X - sbI_*{}^2(4I^2)^{-1}(1 + (\pi k I(g_1 I_*)^{-1})^2)X + 4sI_*{}^2(bI)^{-1}(1 + (\pi k I(g_1 I_*)^{-1})^2)^{-1}Y \tag{24}$$

$$\dot{Y} = -\pi(g_1 I)^{-1}I_* ZX + +(\pi I_*)^2(g_1{}^2 I)^{-1}r_1 X - 0.25b(1 + (\pi k I(g_1 I_*)^{-1})^2)Y \tag{25}$$

$$\dot{Z} = -\pi(g_1 I)^{-1}I_* XY - b(I_* I^{-1})^2 Z. \tag{26}$$

In this case the conductor voltage drop is defined by the following expression:

$$u(t) = u_l(t) + u_r(t) = L_\rho I_0\big(c^2 t_0\big)^{-1}\dot{\phi} + R_\rho I_0(b_1(t)I - \big(b_2(t)Y + b_3(t)Z)r_1{}^{-1}\big) \tag{27}$$

where $\phi = L_\delta I$ is the magnetic flux coupled with the conductor inductance; $L_\delta(t)$, $b_1(t)$, $b_2(t)$ and $b_3(t)$ are coefficients, considering the change of the inductance and the resistance caused by the soliton's increase into the conductor.

3. Discussion

In [1,2] we showed that the dynamical system (equations (12)–(14)) has a subcritical bifurcation. Under this bifurcation, any trajectory of the system in phase space by $r_1 > r_*$ (r_* is the control parameter (r_1) value which corresponds to the stability level; for $I = 1 =$ constant, $r_* = 1.488$) goes to infinity during a finite time. In the case of the direct current this singularity has the form $X = -2.442(t_* - t)^{-1}$ and $Y = Z = -2.442s^{-1}(t_* - t)^{-2}$. Also, in the case of the inductive energy source this singularity has the form $X \sim (t_* - t)^{-1}$, $Y \sim Z \sim (t_* - t)^{-3/2}$ and $I \sim (t_* - t)^{-1/2}$. (The time characteristic quantity 't_*' can be called an explosion point because this transition is called explosive [9].) In both cases the power '$P = U_r I$', which dissipated into the exploding conductor, behaves as $P \sim (t_* - t)^{-2}$.

The above obtained set of (23)–(25) agrees accurately with the set (12)–(14) by $I = I_*$ and not-too-big values of $\mathrm{d}I/\mathrm{d}t$. As this takes place, the bifurcation type is conserved. The initial conditions for (23)–(25) are defined by (12)–(14). However, as this takes place, an indeterminacy of the transition time 't_r' from one set to the other has evolved. The singular behaviour of amplitudes X, Y, Z and I are defined by the expressions $I \sim (t_* - t)^{1/2}$ and $X \sim Y \sim Z \sim (t_* - t)^{-1/2}$. Obviously, the electric current is not singular, but the behaviour of the electric current derivative is singular: $\dot{I} \sim (t_* - t)^{-1/2}$. Since, the behaviour of the conductor voltage drop is defined by the expression $U_r \sim Y \sim Z \sim (t_* - t)^{-1/2}$, the power, which dissipates into the exploding conductor, behaves as $P = U_r I \sim (t_* - t)^0 = \mathrm{O}(1)$. Consequently, the formation of the conductor low-conducting current state as a consequence of the conductor splitting—the conductor stratification or the spatial scale splitting—is energetically more profitable than the generation and overspeeding of the current vortex structures. If this holds, the spatial scale splitting is a first-order non-equilibrium phase transition (which leads to the formation of metallic particles with a size of radius r_0). At the increase of the soliton (the ring slit) convective influx of magnetic field strength $H_* = 2I_*(cr_0)^{-1}$ into the conductor is realized. Towards the end of the conductor stratification the electric current density at its axis is directed to infinity. Obviously, the model of the first stage of the spatial scale splitting obtained above (equations (23)–(25)) loses its validity for $1 + \delta \to 0$ ($\delta \to -1$). Actually the validity of the model breaks down some time before (this is caused by the disregard of the next stages of the spatial scale splitting).

The problem of the identification of model parameters defined by (12)–(14) has been solved, using experiments on the electrical explosion of conductors. (One of these experiments has the following conditions: the exploding conductor is a copper wire 54 μm in diameter and 4.5 cm long; the circuit capacity is 0.486 μF; the circuit inductance is 0.568 μG; the circuit stray resistance is 0.14 Ω, and the charging voltage is 22.144 kV.) This identification has been made to clear up the possibility of practical applications of the above-suggested NPT models and for finding the transition time size from (12)–(14) to (23)–(25). On this occasion, the model suggested in [10] has been used with the description of the heating stage of the exploding conductor. Problems spring up on joining the models and accommodating the calculated results to the experiment. These problems have been caused by the necessity of finding not only the model parameters, but also the initial magnitudes of the X, Y and Z amplitudes. Figure 2 shows results of this accomodation: figure 2(a) shows the conductor voltage drop U_r, and (b) shows the electric current $I(t)$, which was obtained experimentally, and the current vortex structures at characteristic time points; (c) corresponds to the end of the heating stage ($t = 0.301$ μs); (d) corresponds to the maximum value of the electric current ($t = 0.308$ μs), and (e) corresponds to the inflection point of the experimental curve of the electric current ($t = 0.415$ μs). (This point corresponds to the maximum value of the conductor effective resistance.) Figure 2(e) shows that the inflection point corresponds practically to a completely covered channel for the electric current, being closed through the external electric circuit. From this figure we notice that the transition to the model defined by (23)–(25) can be performed within a range of time points, beginning with $t \approx 0.4$–0.415 μs. This indeterminacy has been caused by an information loss exhibited by a reduction of the input (1)–(3) to (23)–(25). This indeterminacy can be diminished if an upper limit is given for the value of the current derivative. The transition to (23)–(25) can be realized as soon as the current derivative remains below this value.

The results are found to be in direct relation to the initial size of X. This is due to the fact that an answer to the question 'to explode or not to explode' is determined not only by the control parameter size, but also by the initial level of excitations as well. On the

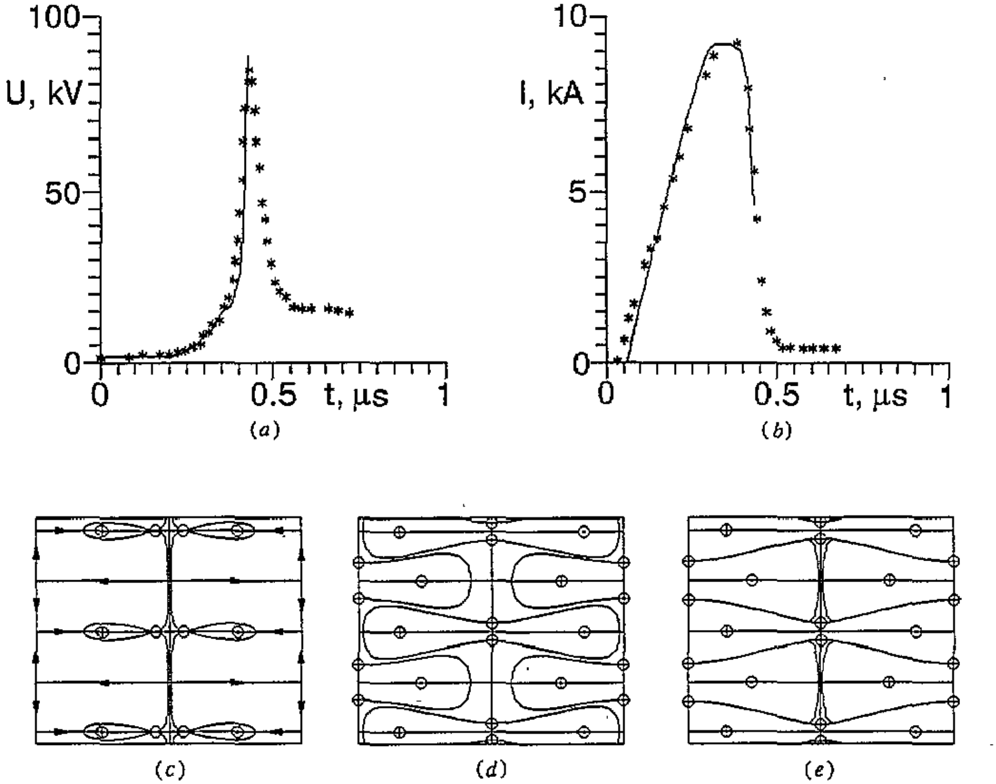

Figure 2. Accommodation results for the model of (12)–(14) from experiments with exploding electrical conductors, (The experimental conditions are described in section 3.) The pattern (*a*) shows the conductor voltage drop; (*b*) shows the electric current (the full curves correspond to theoretical data, and points correspond to experimental ones); (*c*), (*d*) and (*e*) show the current vortex structures for the characteristic time points: (*c*) $t = 0.301$ μs, (*d*) $t = 0.308$ μs and (*e*) $t = 0.415$ μs.

curve of the conductor voltage drop (figure 2(*a*)) a step-like behaviour is clearly seen. This step-like behaviour is caused by the melting of the exploding conductor. Precisely, during this step-like behaviour the current vortex structures are developed (see figures 2(*c*)–(*e*)). The estimate of the shear viscosity, which was made from model parameters which were obtained as a result of the accommodation of the calculated data for the experiment, gave a viscosity value which approaches the table values of the liquid metal's viscosity. This fact counts in favour of the above hypothesis on the local constancy of the transport coefficients.

As further arguments in support of the above model, the direct observation results of the *stratification of copper wire of 0.58 mm diameter can be mentioned.* (The electric circuit has the following parameters: the period of vibration is 40 μs, the capacity is 4.2 μF, and the charging voltage is 30 kV [8] (see also [3]).) These observations show that the time interval between the beginning and the end of the conductor stratification equals 200 ns. This value is essentially less than typical sizes of the sound time. Besides, the experiments for the preparation of the ultra-dispersed powder by the electrical explosion of conductors [11, 12] show that one obtains a large fraction with the size of the conductor diameter. (Its concentration is small, but, in practice, contains all the mass of the EEC products.)

4. Conclusion

In this paper the first-order non-equilibrium phase transition in current-carrying plasma-like fluids has been investigated. The conductor stratification and the electric current interruption—the forming of the heterogeneous low-conducting current state of the conductor—are defined by this model as the interaction between the hydrodynamic velocity field ($\boldsymbol{u}(\boldsymbol{r}, t)$) and the magnetic field ($\boldsymbol{H}(\boldsymbol{r}, t)$). In this way, the formation of the low-conducting state through the splitting of the spatial scale of hydrodynamic vortex structures—the first-order non-equilibrium phase transition—is energetically more profitable than through the excitation and overspeeding of current vortex structures—the second-order non-equilibrium phase transition.

Acknowledgments

We are grateful to K E Bobrov for his assistance in producing the computer drawings for this paper. We also greatly appreciate the assistance of Drs T A Golub and N A Gondarenko in solving the problem of the identification of the model parameters. We would like to express our gratitude to Mrs A D Neifeld for her help in preparing the English version of the paper. We also want to express our deep gratitude to the Russian Foundation for fundamental investigations for financial support of this work (grant N 94–02–06654-a).

References

[1] Volkov N B and Iskoldsky A M 1993 *J. Phys. A: Math. Gen.* **26** 6635
[2] Volkov N B and Iskoldsky A M 1993 *J. Phys. A: Math. Gen.* **26** 6649
[3] Volkov N B and Iskoldsky A M 1993 *J. Phys. A: Math. Gen.* **26** 6667
[4] Volkov N B and Iskoldsky A M 1990 *JETP Lett.* **51** 634
[5] Saltzman B 1962 *J. Atmos. Sci.* **19** 329
[6] Lorenz E N 1963 *J. Atmos. Sci.* **20** 130
[7] Lamb H 1932 *Hydrodynamics* (Cambridge: Cambridge University Press)
[8] Iskoldsky A M 1985 *Thesis* (High Current Electronics Institute: Tomsk) (in Russian)
[9] Richtmyer R D 1981 *Principles of Advanced Mathematical Physics* vol 2 (New York: Springer)
[10] Golub T A, Gondarenko N A and Iskoldsky A M 1989 *Proc. Int. Workshop on the Physics and Technology of High Power Electric Current Interrupters* (Novosibirsk) p 80
[11] Weber F N and Shear D D 1969 *J. Appl. Phys.* **40** 3854
[12] Yavorovsky N A 1982 *Thesis* (Polytechnic Institute: Tomsk) (in Russian)